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On the relevance of tachyons

Jeffrey A. Harvey[a], David Kutasov[b] *and* Emil J. Martinec[c]

Enrico Fermi Inst. and Dept. of Physics
University of Chicago
5640 S. Ellis Ave., Chicago, IL 60637, USA

We study condensation of open string tachyons using renormalization group flow in the worldsheet field theory. This approach leads to a simple picture of the physics of the nontrivial condensate.

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[a] harvey@theory.uchicago.edu

[b] kutasov@theory.uchicago.edu

[c] e-martinec@uchicago.edu

The configuration space of open string theory is of intrinsic interest; it is the arena for D-brane dynamics, and may teach us about other nonperturbative string phenomena. An example of such a phenomenon that has received some attention recently [1-16] is tachyon condensation in unstable D-brane systems. It has been proposed [2] that the endpoint of the condensation is the closed string vacuum.

An approach using level truncation of open string field theory yields remarkably accurate results for quantities such as the vacuum energy [1,10,11,14,15] and the properties of low-lying excitations [13,16] of the nontrivial condensate, which appear to support these ideas. However, it is not easy to understand why this should be so within the context of string field theory. Also, it is difficult to study the properties of the non-trivial vacuum using this approach.

In this note, we point out that the worldsheet boundary renormalization group provides a simple conceptual framework for understanding open string tachyon condensation. We will see that an analysis of the RG flows induced by boundary perturbations yields a qualitative and quantitative physical picture of the open string configuration space. This makes it possible to find non-trivial classical solutions and study excitations around them.

In particular, using results in the literature one can prove that unstable branes indeed annihilate by tachyon condensation, compute the energy of the vacuum, and show that all open string modes disappear from the spectrum in the process. One can also study flows in which an unstable brane (or brane-antibrane pair) annihilates into any number of lower dimensional branes.

The bosonic string

String dynamics is succinctly specified as the condition of Weyl invariance on the string worldsheet Σ. The classical open string equations of motion are the vanishing of the beta functions for boundary perturbations of the two-dimensional action on the disk. Flowing to the IR on the worldsheet is equivalent to approaching a classical solution of the spacetime theory. Thus the study of tachyon condensation in open string theory is equivalent to an examination of the RG flow induced by boundary perturbations, for example

$$\int_{\partial\Sigma} d\tau\, T[X(\tau)] \tag{1}$$

in the open bosonic string, with $T[X]$ slowly varying in spacetime.

To begin, consider the boundary RG flow with a single cosine potential

$$T[X_{25}] = -\lambda \cos[kX_{25}] \tag{2}$$

i.e. the boundary Sine-Gordon model. We choose conventions such that $\alpha' = 1$. Thus $k^2 < 1$ for a relevant perturbation; we restrict our discussion to this situation. The theory with boundary interaction (1) is exactly solvable [17] using Bethe ansatz techniques. The UV fixed point is free field theory with Neumann boundary conditions. The coupling λ increases under flow to the IR, pinning the boundary value of the field to one of the minima of the potential, which are spaced a distance $\delta X_{25} = 2\pi/k$. Thus, the IR fixed point is a stack of D24-branes.

Under boundary RG flow, the trace of the stress-energy tensor is only nonzero on the boundary; the bulk theory remains conformal, and in particular the central charge c is fixed. Thus the flow is in the space of boundary conditions of a given bulk theory. It is believed that there is a quantity analogous to c, which measures the 'number of boundary degrees of freedom', and decreases along RG flows. This quantity is the boundary entropy g [18], which can be defined as the term in the annulus partition function which is independent of the width L of the strip in the thermodynamic limit $L \to \infty$

$$\beta\mathcal{F} = \beta\mathcal{F}_{\text{bulk}} - \log[g] \ , \tag{3}$$

or equivalently as the disk partition function $g_B = \langle 0|B\rangle$ of the boundary state $|B\rangle$ associated to the perturbed theory. Affleck and Ludwig [18,19] showed that g decreases along RG flows at the lowest nontrivial order in conformal perturbation theory, but there is as yet no proof that this is always the case.

Quite generally, g measures the tension of the corresponding D-brane in string theory [20]; physically, the g-conjecture should be related to minimization of the action in the space of open string fields.

In any event, for the boundary Sine-Gordon interaction one has (from the exact solution using the thermodynamic Bethe ansatz [17])

$$\frac{g_{\text{IR}}}{g_{\text{UV}}} = k \ . \tag{4}$$

This result agrees with the ratio of energy densities of the IR stack of D24-branes relative to the UV D25-brane. Indeed, assuming that X_{25} is compactified on a large circle of radius R, the boundary entropy is[1] [17,21]:

$$\begin{aligned} g_{\text{UV}} &= \sqrt{R} \\ g_{\text{IR}} &= kR \cdot \frac{1}{\sqrt{R}} \end{aligned} \tag{5}$$

[1] A rescaling is needed to relate g_{UV} and g_{IR} to the tensions of the corresponding branes [20].

where kR is the number of D24-branes at the IR fixed point of the flow (2). $g_{\rm IR}/g_{\rm UV}$ computed from (5) agrees with the exact calculation (4); it is also consistent with the conjectured "g-theorem", since $g_{\rm IR}/g_{\rm UV} < 1$ precisely when the perturbation (2) is relevant.

In the case $k^2 = 1$ the perturbation (2) is truly marginal and the coupling λ parametrizes a line of fixed points, interpolating between Neumann (small λ) and Dirichlet (large λ) boundary conditions [22,23]. The boundary entropy g is independent of λ, as one would expect from the g-conjecture.

The RG flow of the interaction (1) has also been studied using conformal perturbation theory [24]. While TBA techniques establish the exact result for the IR boundary entropy (4), it is useful to compare the conformal perturbation theory with the method of level truncation in string field theory [1,10]. The latter appears to converge quite rapidly. Conformal perturbation theory is simply the expansion of the partition function in a power series in λ

$$\mathcal{Z} = \sum_{n=0}^{\infty} \lambda^{2n} \mathcal{Z}_{2n} \ , \tag{6}$$

where the coefficients $\mathcal{Z}_{2n}$ are given by the integrated correlation functions of $2n$ of the tachyon perturbations (1). Bounds on the $\mathcal{Z}_{2n}$ establish that the radius of convergence of the expansion is infinite for $k^2 < \frac{1}{2}$ [24]. Expanding the boundary entropy $\log[g] = -\partial\mathcal{F}/\partial T$ in λ, the series converges quite rapidly; for example, $\log[g_{\rm IR}]$ is obtained to within 4% for $k^2 = \frac{1}{3}$, keeping only the first seven terms in the expansion [24]. Thus conformal perturbation theory provides a viable alternative to the level truncation approach in string field theory as an approximation to the dynamics.

The limit $k \to 0$ isolates a single D24-brane, if we remain at a finite point in space; alternatively, if we perturb by $\sin(kX_{25})$, the branes are pushed off to infinity as $k \to 0$, and we are left with the closed string vacuum as conjectured in [2]. In the limit, the term in the energy proportional to the volume vanishes, since $g_{\rm IR}/g_{\rm UV} \to 0$.

One might wonder what has happened to the dynamics of open strings as a result of the tachyon condensation. From the worldsheet point of view, all boundary operators $\exp(ipX_{25})$ are flowing to the identity operator under the RG flow (they become c-numbers when the boundary conditions on X_{25} are Dirichlet); X_{25} becomes localized at one of the minima of the potential (2). In spacetime, normalizable eigenstates of the Hamiltonian in the unstable vacuum $T = 0$ (corresponding to the UV fixed point of (2)) are wavepackets which move freely in X^{25}; at the IR fixed point, all the normalizable modes are bound to the D24-brane, as was found for the lowest modes in [13]. In addition, since the kinetic terms for these spacetime fields are given by the L_0 eigenvalue of the corresponding worldsheet operator, these terms vanish in the IR as suggested in [9].

The worldsheet analysis is also consistent with the discussion of ref. [1], where evidence was presented that after the tachyon condenses, the inverse propagator of some low lying fields no longer has any zeroes, which means that they do not give rise to physical excitations (in the context of our present discussion, the analysis of [1] describes the region far from the D24-branes). The worldsheet picture shows that this is true for all modes of the open string, as one would expect.

It is important to emphasize that this process of binding of physical excitations to a wall is entirely classical in string theory. A breakdown of the classical approximation would manifest itself as a singularity in the RG flow corresponding to (1), (2). Such a singularity certainly does not exist for the flow (2) and there is no reason to expect one for the more general case (1). Note also that the above discussion is true in particular for the open string gauge field, and for any fields that couple to it. The disappearance of the gauge field dynamics in the bulk is thus a *classical* phenomenon.

At finite k, a finite spacing between D24-branes is maintained at the IR fixed point. There should be open string sectors corresponding to the stretched strings between different branes; how do these arise? At any point along the flow, the cosine potential will have solitons on the boundary, where the boundary value of X_{25} hops from one minimum of the cosine to another. The soliton creation and annihilation operators will flow to the vertex operators that create and destroy the corresponding stretched strings at the IR fixed point. These solitons descend from highly excited open string states. In the typical such state, the configuration of the open string is a random walk [25]; the cosine potential then draws the endpoints to the minima at $X_{25} = 2\pi n/k$, but not necessarily the same one at both endpoints. The lightest stretched strings presumably descend from the leading Regge trajectory $(\partial X_{25})^N$ of the UV fixed point, which generates a straight stretched string of length $\sim \sqrt{N}$ in string units.

We next turn to the case of a general slowly varying tachyon condensate $T(X_{25})$ depending only on X_{25}. Generically, T will have a series of minima at some points X_i, near which it behaves like $T(X) \sim (X - X_i)^2$. The infrared fixed point of (1) will in this case describe a series of D24-branes located at the points X_i. One can ask what happens when different minima coincide and one considers a multicritical potential $T(X) \sim (X - \hat{X})^{2m}$, near some point $\hat{X}$. If $T(X)$ were a bulk perturbation, the system would flow in the IR to the $(m+1, m+2)$ minimal model, with central charge $c = 1 - \frac{6}{(m+1)(m+2)}$ [26]. The multicritical boundary potential appears instead to describe in the IR a free CFT with $U(m)$ Chan-Paton factors.

As different X_i approach each other, strings stretched between different minima of T go to zero mass (or worldsheet scaling dimension), and in the limit, one finds an m^2-fold degeneracy of all operators. A priori, it is not obvious that the sole effect of this degeneracy is to introduce Chan-Paton factors – the system could in principle be interacting in the IR, but the picture of coalescing D-branes suggests that it is in fact free.

The above discussion is consistent with [27], where it was shown that the ratio of the boundary entropies resulting from a flow $X^{2m} \to X^{2m-2}$ in the boundary interaction is

$$\frac{g_{\rm IR}}{g_{\rm UV}} = \frac{m-1}{m} \ . \tag{7}$$

There is also some evidence that the multicritical boundary theories with potential $T(X) \sim X^{2m}$ are related to the underscreened Kondo model with spin $(m-1)/2$, which has the same boundary entropy [28]. This relation might help in making the connection with Chan-Paton dynamics, but we are not going to pursue this issue here.

From the structure of the tachyon potential in open string field theory, $V(T) \sim -\frac{1}{2}T^2 + bT^3$, (valid for small T) it might appear that there is a dramatic difference in the dynamics for opposite signs of T. However, from the worldsheet perspective, it is clear that perturbations by $\pm T$ lead to qualitatively the same physics. As discussed above, turning on a perturbation (1) leads in the IR to physics dominated by the minima of the potential $T(X)$. If we take $T \to -T$, we find similar physics, with the degrees of freedom localized at the maxima of T.

For example, taking X to be compact and turning on a slowly varying tachyon field, which is an arbitrary combination of relevant boundary operators, one finds both for T and for $-T$, a maximum number $[R]$ of D24-branes which can move around in X^{25} and annihilate. Of course, in agreement with perturbative string theory there is no $T \to -T$ symmetry, as the properties of the minima and maxima of T, for a generic given T, are not the same.

The apparent instability of the spacetime potential $V(T)$ for negative T is reflected in the worldsheet approach as the fact that if $T(X)$ (for non-compact X) is slowly varying but not bounded from above, $-T$ is not bounded from below, and the worldsheet theory appears to be sick. From the general perspective it is clear that one should think of this instability as describing a D24-brane (that is a minimum of T) that was pushed to $|X| \to \infty$.

So far we discussed tachyon profiles that depend only on X_{25}. The D24-branes obtained in the infrared clearly have a tachyon on their worldvolume which can condense further to make lower D-branes. The general tachyon condensate depending on all twenty-five spatial coordinates leaves some set of isolated D0-branes; Dp-branes are simply metastable fixed points of the flows.

One can also study time-dependent tachyon profiles. To use results from boundary field theory, one should presumably Wick-rotate to Euclidean time and repeat the previous discussion. As before, in the IR the worldsheet dynamics is localized at a particular value of X_0. This value might be interpreted as the time of annihilation of the unstable D-brane.

The fermionic string

In the type II superstring, there are unstable non-BPS Dp branes with even p for type IIB and odd p for type IIA (for a review, see [8]). Sen has shown [4,5] that the stable, BPS $D(p-1)$ brane can be described as a tachyonic kink soliton solution of the spacetime effective field theory (the RR charge of the kink arises due to an $\int C_p \wedge dT$ coupling in the effective action). We can again describe this situation via a perturbation of the worldsheet field theory by a boundary superpotential

$$S_{\rm bdy} = \int_{\partial\Sigma} d\tau \left[\eta \frac{d\eta}{d\tau} + \eta(\tau) \int d\theta \; T[\mathbf{X}_p] \right] . \qquad (8)$$

Here, $\mathbf{X}_p$ is an N=1 worldsheet superfield, and η is an anticommuting quantum-mechanical degree of freedom living on the boundary, which is needed in order that the tachyon vertex operator describes a spacetime boson [29]. Equivalently, one can add 2×2 Chan-Paton factors to the vertex [8]. In this case, both maxima and minima of the tachyon profile T contribute supersymmetric minima of the worldsheet boundary potential $(T')^2$, leading to codimension one branes.

In the IR, the potential acts as a mass term that decouples η, and turns the boundary condition on $\mathbf{X}_p$ from Neumann to Dirichlet. One cannot write a tachyon perturbation of the IR fixed point because the extra boundary fermion η needed to write it is absent (similarly for all the other GSO-odd vertex operators); thus the infrared theory at an extremum of the boundary potential is GSO projected. The maxima and minima of T give opposite contributions to $\mathrm{tr}(-1)^F$ in the boundary quantum mechanics, so the corresponding boundary states will have opposite signs in the contribution of the RR sector (periodic fermion boundary conditions in the closed string channel are equivalent to inserting $\mathrm{tr}(-1)^F$ in the open string channel). In other words, the minima give (say) $D(p-1)$ branes (boundary state $|NSNS\rangle + |RR\rangle$), while the maxima yield $\overline{D}(p-1)$ branes (boundary state $|NSNS\rangle - |RR\rangle$).

One can generalize the discussion of (4) to this case by turning on a superpotential $T = \lambda \cos(k\mathbf{X}_p)$. There is no analog of the exact solution [17] for this case, but one can still compute the change in the boundary entropy using the brane picture and check consistency with the g-conjecture.

The tensions of the relevant branes are

$$\begin{aligned} \mu_p^{(\mathrm{un})} &= \frac{\sqrt{2}}{g_{\mathrm{str}}} \cdot (2\pi)^{-p} \\ \mu_{p-1}^{(\mathrm{st})} &= \frac{1}{g_{\mathrm{str}}} \cdot (2\pi)^{-(p-1)} \end{aligned} \tag{9}$$

where $\mu_p^{(\mathrm{un})}$ and $\mu_{p-1}^{(\mathrm{st})}$ are the tensions of the unstable p-brane and stable (BPS) $(p-1)$-brane, respectively. The spacing of the alternating stack of $D(p-1)$ and $\overline{D}(p-1)$ branes is $\frac{1}{2} \cdot (2\pi/k)$, so

$$\frac{g_{\mathrm{IR}}}{g_{\mathrm{UV}}} = \frac{2kR\, \mu_{p-1}^{(\mathrm{st})}}{2\pi R\, \mu_p^{(\mathrm{un})}} = \sqrt{2}\, k\ . \tag{10}$$

Relevance of the superpotential T implies that $k^2 < 1/2$ in this case; hence, the result (10) is again consistent with the g-conjecture.

The disappearance of the kinetics in the p^{th} direction is the same as in the bosonic string. The worldsheet analysis shows that all the open string fields are bound to a lower-dimensional brane after the tachyon condenses. There are no normalizable excitations of the open string fields in the bulk. The effect is again classical.

It is easy to generalize the above analysis to the boundary flow corresponding to the generation of a non-BPS $D(p-1)$ brane as a tachyonic kink on a nearby $Dp - \overline{D}p$ pair. The calculation (10) works in a similar fashion. One finds a non-GSO projected spectrum in the IR in this case as a consequence of the fact that the analog of η (8) is complex in the $Dp - \overline{D}p$ system. One combination of η, η^* becomes massive in the flow to the IR, while the other remains massless and allows one to construct excitations with odd worldsheet fermion number like the tachyon on the non-BPS $(p-1)$-brane. One can also use boundary RG to study the decay of non-BPS states on K3 [6] (and other compactifications) as one varies the closed string moduli of the compactification.

The discussion of a general superpotential $T(X)$ (8) is very similar to the bosonic case. One difference is that there is no longer any instability; the bosonic potential $(T')^2$ is always bounded from below. In fact, the RG flow corresponding to (8) has a manifest symmetry, which takes $T \to -T$ (exchanging minima and maxima of T and thus, in the worldsheet IR limit, BPS $D(p-1)$ and $\overline{D}(p-1)$ branes) and flips the sign of all the RR fields (thus exchanging D and $\overline{D}$ branes back).

Discussion

One may wonder what is the precise relation between boundary RG flows and open string field theory. In particular, why do calculations in the level-truncated string field theory [1,10,11,14,15] converge so rapidly to the right answer for the vacuum energy? We close with some speculations in this regard.

There are strong analogies between string field theory equations of motion and the exact (Wilsonian) RG [30]. One studies the RG flows of the theory with a finite cutoff and all possible operators in the action. In this interpretation, the Wilsonian RG equations are thought of as the Langevin equation of stochastic quantization [31] of the string field theory. RG scale is stochastic time, in which the couplings relax toward an extremum of the action. The complete set of couplings includes operators involving the boundary curvature and its derivatives; in the 'conformal normal ordering' prescription of Polchinski [32], these are encoded in couplings to the reparametrization ghosts. Formally, the space of interacting string fields considered in [7,10] can be mapped onto the space of general boundary perturbations using the formalism of [33] combined with Polchinski's prescription. Thus we expect that open string field theory represents a formulation of the exact boundary RG in a very particular renormalization scheme.

One might then interpret the approximation of adding more and more levels of the string field as the standard procedure of RG improvement by adding small amounts of irrelevant operators to the cutoff theory in order to more closely approximate the continuum limit. Typically such a procedure does well even when only the first few irrelevant terms are included; the effects of highly irrelevant operators are highly damped by their rapid decay. This may provide a qualitative explanation of the rapid convergence of the string field theory calculations of [1,10,15].

Another natural set of questions surrounds closed string tachyon condensation. Following the logic of this paper, turning on a closed string tachyon leads to lower-dimensional vacua, characterized by the decrease of c along the flow [26], and the localization of physical excitations in space. In the bosonic string, two dimensional string theory (*c.f.* [34]) is a possible endpoint of the condensation. In this stable vacuum, the whole closed string is confined to 1+1 dimensions (unlike the open case, where only the endpoints are confined); all the transverse oscillator towers of the string are frozen in the new vacuum.

The spacetime description of this drastic reduction in the number of degrees of freedom is very similar to that of the open string. The normalizable excitations of the string center of mass are bound to a lower dimensional manifold. Unlike the open string case, the closed string oscillator degrees of freedom don't have any normalizable excitations.

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